Supplementary information for

**A simple mechanical hypothesis for the diversity and speed-dependency of the fast gaits of animals**

DOI to be confirmed

Author to be confirmed

*Additional Supplementary information:*

Powerpoint ‘Animations’ with .gifs showing development from forces through accelerations and velocities to powers relating to Figure SF2.

Excel spreadsheet ‘Example_Calculation_Spreadsheet’ (importable to Google Sheets) demonstrating the process of numerical integration for calculating the power demands of fast four-legged gaits.

Zipped folder ‘Code’ containing the LabView code used for the calculations shown in the paper and animations in the Powerpoint Supplementary information.

*Supplementary information here:*

Figure SF1. Gait description decision tree, an extended version of Figure 1.

Figure SF2. Graphical representation of the process of calculating positive mechanical work.

Table ST1. Summary of terms required and results for each gait for the impulsive work demand model (Section 2.1).

Table ST2. Summary of conditions allowing theoretical zero mechanical work demand.

Extended power calculations.

Extended glossary.

Figures SF1 and SF2 in editable PDF format.

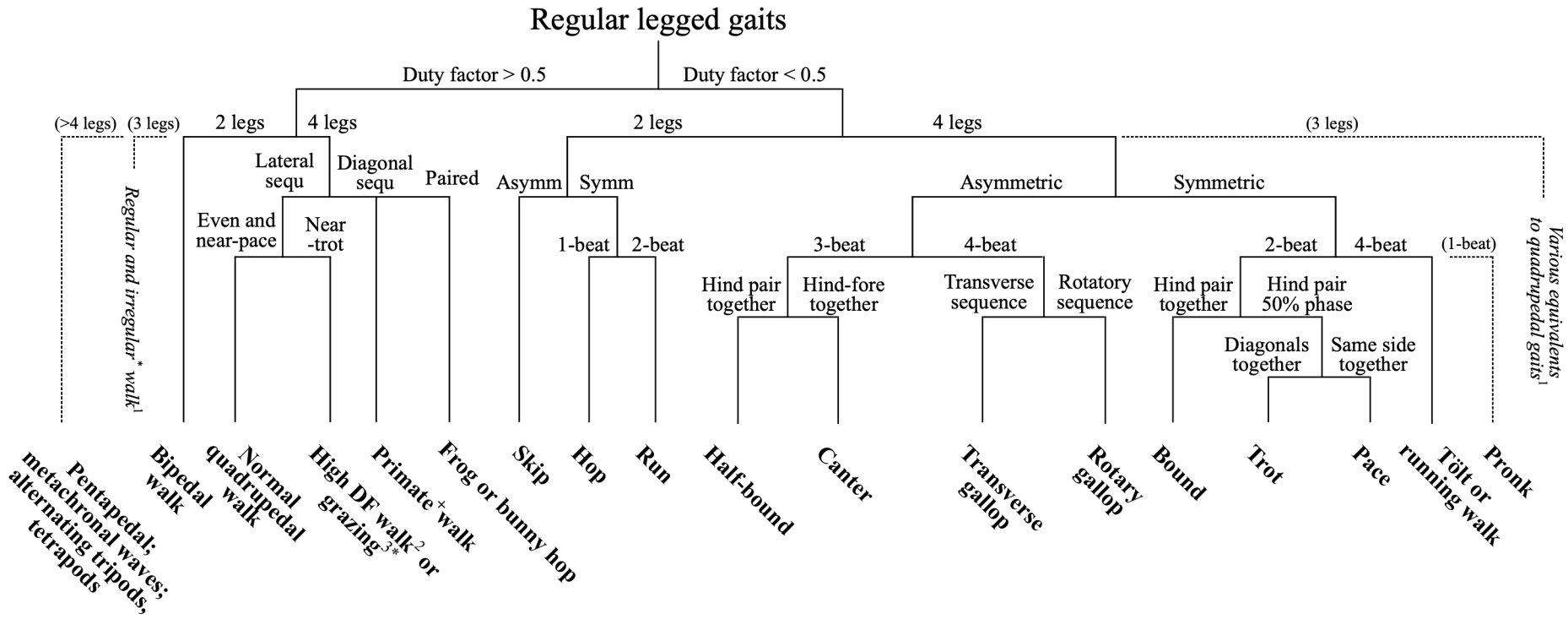



Figure SF1. Overview of gait descriptions of legged terrestrial animals largely following conventions and definitions of Gray, 1968 and Hildebrand, 1989. 'Together' refers to timing; for instance, the canter is a three-beat gait with simultaneous contact of one of the hind and one of the forefeet. Symmetric gaits have an even interval between left and right contacts. *Grazing may be quite irregular, tripedal walk can be even more so, sometimes with different hind- and fore- step frequencies. [+]The diagonal sequence walk is typical but not universal across primates and is observed in a range of other species, often related to walking along branches. Lateral and diagonal sequence walks begin with a hind foot: if lateral sequence beginning Left Hind, then Left Hind (LH), Left Fore (LF), Right Hind (RH), Right Fore (RF); if transverse sequence LH, RF, RH, LF.

**References for gait descriptions**

*General*

Gray, S. J. (1968). *Animal locomotion*. New York: Norton.

Hildebrand, M. (1989). The quadrupedal gaits of vertebrates. *Bioscience*, *39*(11), 766.

*Niche*

[1] Self Davies, Z. T., Savage, A. L., & Usherwood, J. R. (2026). Locomotion on three legs: the tripedal gaits of canine amputees. *Proceedings of the Royal Society B: Biological Sciences*, *293*(2069).

[2] Usherwood, J. R., & Self Davies, Z. T. (2017). Work minimization accounts for footfall phasing in slow quadrupedal gaits. *Elife*, *6*, e29495.

[3] Usherwood, J. R., & Smith, B. J. (2018). The grazing gait, and implications of toppling table geometry for primate footfall sequences. *Biology Letters*, *14*(5), 20180137.

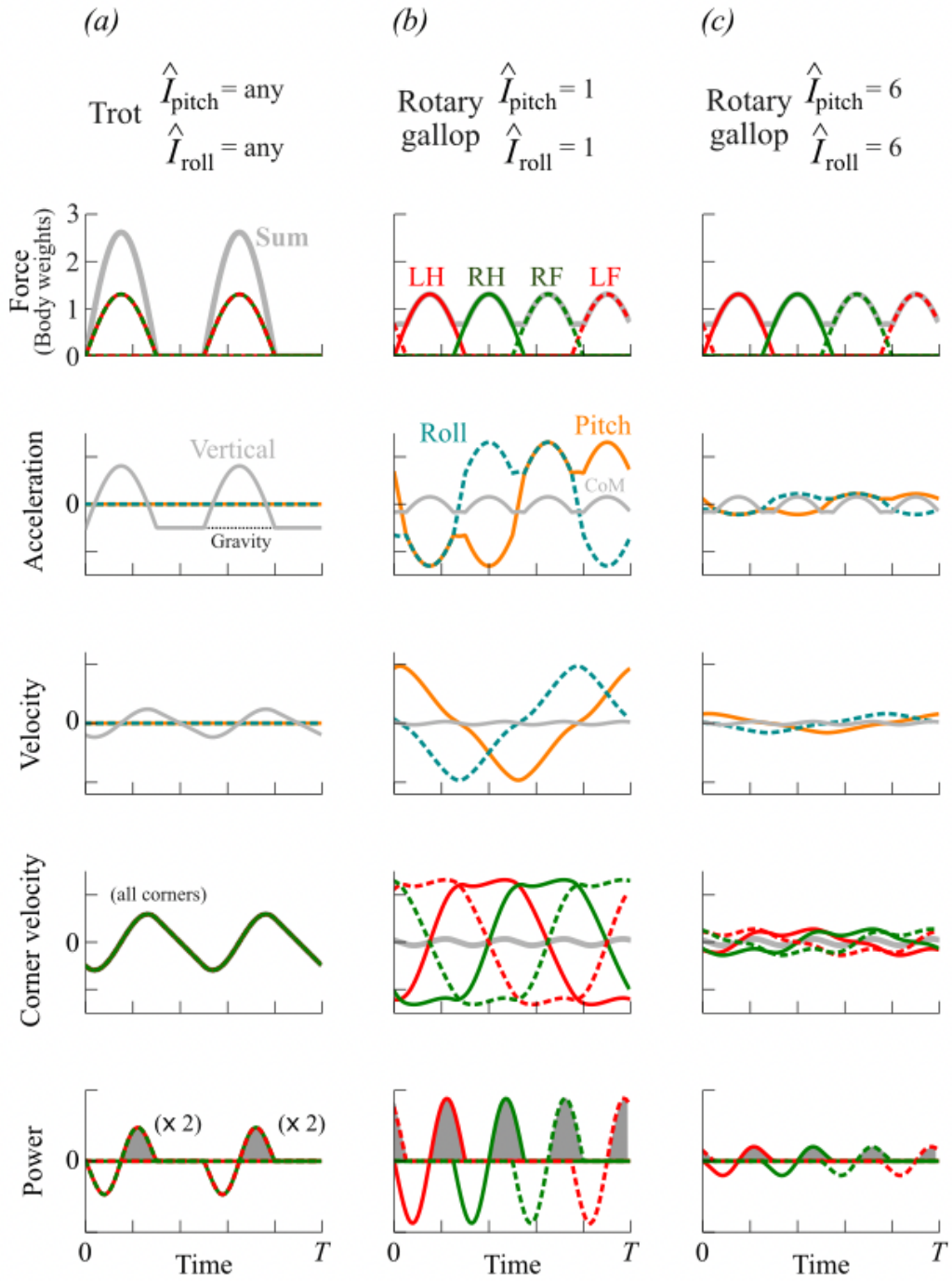



Figure SF2.
Graphical representation of the process of calculating positive mechanical work (grey shaded areas) demand for trot (*a*), rotary gallop with low moments of inertia (*b*) and rotary gallop with high moments of inertia (*c*) at a duty factor of 0.3. Trot imposes no angular accelerations, but results in high fluctuations in centre of mass velocity. Rotary gallop with low moments of inertia result in low centre of mass CoM accelerations and velocities, but high angular accelerations and velocities, resulting in high velocities of each corner when loaded, and so high mechanical power demands; trotting would have been favoured. With high moments of inertia, the low centre of mass accelerations and velocities, and reduced angular accelerations and velocities, result in low corner velocities and so low power demands; a rotary gallop would be favoured. See Electronic Supplementary Information Excel file and Extended Power Calculation section.

| Terms | Symbols or normalisation | Biped | | Quadruped | | | |
|---|---|---|---|---|---|---|---|
| | | Hop | Run | Trot | Transverse gallop | Tölt | Rotary gallop |
| Number of vertical impulses each stride | $N_{CoM}$ | 1 | 2 | 2 | 4 | 4 | 4 |
| Number of rotational impulses each stride | $N_{rot}$ | 0 | 2 (L, R) | 0 | 4 (L, R, L, R) | 4 (H, F, H, F) | 4 (LH, RH, RF, LF) |
| Rotational moment arm | $r$ | N/A | $w/2$ | N/A | $w/2$ | $L/2$ | If $w=L$, $\frac{L}{\sqrt{2}}$ |
| Moment of inertia | $I$ | N/A | $I_{roll}$ | N/A | $I_{roll}$ ($I_{pitch} \approx \infty$) | $I_{pitch}$ ($I_{roll} \approx \infty$) | $I_{diag} = I_{roll} = I_{pitch}$ |
| Sum work demand on CoM | $\frac{\sum W^{+}_{CoM}}{mg^2T^2/16}$ | 2 | 1 | 1 | ½ | ½ | ½ |
| Sum rotational work demand | $\frac{\sum W^{+}_{rot}}{mg^2T^2/16\hat{I}}$ | 0 | 1 | 0 | ½ | ½ | 1 |
| Conditions for energetic advantage | | $\hat{I}_{roll} < 1$ | $\hat{I}_{roll} > 1$ | $\hat{I} < 1$ | $\hat{I}_{roll} > 1$ and $\hat{I}_{pitch} \approx \infty$ | $\hat{I}_{pitch} > 1$ and $\hat{I}_{roll} \approx \infty$ | $\hat{I}_{pitch} \approx \hat{I}_{roll} > 2$ |

Table ST1. Summary of terms required and results for each gait for the impulsive work demand model (Section 2.1).

| | Biped | | Quadruped | | | |
|---|---|---|---|---|---|---|
| Conditions | Hop | Stride | Walk | (Slow) Trot | (Slow) Bound | 4-beats (Tölt, gallops) |
| Duty factor $\beta$ | x | $\beta \geq \frac{1}{2}$ | $\beta \geq \frac{3}{4}$ | $\beta \geq \frac{1}{2}$ | $\beta \geq \frac{1}{2}$ | $\beta \geq \frac{1}{4}$ |
| $\hat{I}_{\text{roll}}$ | x | $\hat{I}_{\text{roll}} = \infty$ | $\hat{I}_{\text{roll}} =$ any | $\hat{I}_{\text{roll}} = \infty$ or $L_{\text{step}}=L_{\text{foot}}$ | $\hat{I}_{\text{roll}} =$ any | $\hat{I}_{\text{roll}} = \infty$ |
| $\hat{I}_{\text{pitch}}$ | x | $\hat{I}_{\text{pitch}} = \infty$ | $\hat{I}_{\text{pitch}} =$ any | $\hat{I}_{\text{pitch}} =$ any | $\hat{I}_{\text{pitch}} = \infty$ | $\hat{I}_{\text{pitch}} = \infty$ |
| | | | | | | |
| Idealised example | x | Jacana | Tortoise | Alligator | None? | Fast horse, cheetah, elephant |

Table ST2. Summary of conditions allowing theoretical zero mechanical work demand given ideal vertical ground reaction force profiles (Section 2.2).

**Extended power calculations**

If the vertical impulse $J_{\text{limb}}$ of each limb of a quadruped supports its share of body weight $mg$ over the course of the stride $T$ over a half-sine force profile of amplitude $A_z$ over a stance period $\beta T$:

$$J_{\text{limb}} = \frac{mgT}{4} = \int_0^{\beta T} A_z \sin\left(\frac{\pi t}{\beta T}\right) dt, \quad \text{(S1)}$$

meaning that

$$A_{\mathbf{z}} = \frac{\pi mgT}{8\beta T}. \quad \text{(S2)}$$

From this, the vertical force profile over a stance can be modelled for a given duty factor:

$$F_{\mathbf{z,t}} = A_{\mathrm{z}} \sin\left(\frac{\pi t}{\beta T}\right) \quad \text{(S3)}$$

until

$$t = \beta T. \quad \text{(S4)}$$

after which $F_{z,t}$=0. The timing of the vertical force profile due to each limb (Left Hind LH, Right Hind RH, Left Fore LF and Right Fore RF) is determined by the limb phasing, using LH as the start/end of the stride cycle. This, and all subsequent steps, are applied numerically (see supplementary information 'Code' for LabView version, or 'Example_Calculation_Spreadsheet'). The combined force on the centre of mass $F_{\text{zCoM,t}}$ is due to the sum of all the limb forces at each instant:

$$F_{\text{zCoM,t}} = F_{\text{zLH,t}} + F_{\text{zRH,t}} + F_{\text{zLF,t}} + F_{\text{zRF,t}}. \quad \text{(S5)}$$

The roll moment at each instant $Q_{\text{roll,t}}$, using positive for right-up, depends on the forces from each limb and their moment arms about the centre of mass due to the body width $w$:

$$Q_{\text{roll,t}} = \left(F_{\text{ZLH,t}} + F_{\text{ZLF,t}}\right)\left({}^{-w}/_{2}\right) + \left(F_{\text{ZRH,t}} + F_{\text{ZRF,t}}\right)\left({}^{+w}/_{2}\right). \quad \text{(S6)}$$

The pitch moment at each instant $Q_{\text{pitch,t}}$, using positive for nose-up, depends on the forces from each limb and their moment arms about the centre of mass due to the body length $L$:

$$Q_{\text{pitch,t}} = \left(F_{\text{ZLH,t}} + F_{\text{ZRH,t}}\right)\left({}^{-L}/_{2}\right) + \left(F_{\text{ZLF,t}} + F_{\text{ZRF,t}}\right)\left({}^{+L}/_{2}\right). \quad \text{(S7)}$$

The vertical acceleration of the centre of mass at each instant $a_{\text{zCoM,t}}$ is due to the combined forces from the limbs, resisted by the inertia (mass) of the centre of mass, and gravity:

$$a_{\text{zCoM,t}} = \frac{F_{\text{ZCoM,t}}}{m} - g. \quad \text{(S8)}$$

The roll acceleration at each instant $\dot{\omega}_{\mathrm{roll,t}}$ is due to the roll moments, resisted by the roll moment of inertia:

$$\dot{\omega}_{\mathrm{roll,t}} = \frac{Q_{\mathrm{roll,t}}}{I_{\mathrm{roll}}}. \tag{S9}$$

The pitch acceleration at each instant $\dot{\omega}_{\mathrm{pitch,t}}$ is due to the pitch moments, resisted by the pitch moment of inertia:

$$\dot{\omega}_{\mathrm{pitch,t}} = \frac{Q_{\mathrm{pitch,t}}}{I_{\mathrm{pitch}}}. \tag{S10}$$

The changes in vertical velocity of the centre of mass at time $t$ from the start of each stride $\Delta V_{\mathrm{ZCoM,t}}$ to the end (at $t$=T) is calculated through numerical integration without an integration constant (or constant=0):

$$\Delta V_{\mathrm{ZCoM,t}} = \int_0^t a_{\mathrm{zCoM,t}}\ dt. \tag{S11}$$

Assuming, on average, no change in height over the stride, the vertical velocity $V_{\mathrm{ZCoM,t}}$ can be calculated by subtracting the mean value of $\Delta V_{\mathrm{ZCoM,t}}$ over the stride. This method is effectively calculating and removing a suitable integration constant to ensure level locomotion.

Similarly, the changes in roll and pitch angular velocity $\Delta\omega_{\mathrm{roll,t}}$ and $\Delta\omega_{\mathrm{pitch,t}}$ are

$$\Delta\omega_{\mathrm{roll,t}} = \int_0^t \dot{\omega}_{\mathrm{roll,t}}\ dt, \tag{S12}$$

and

$$\Delta\omega_{\mathrm{pitch,t}} = \int_0^t \dot{\omega}_{\mathrm{pitch,t}}\ dt \tag{S13}$$

respectively, and $\omega_{\mathrm{roll,t}}$ and $\omega_{\mathrm{pitch,t}}$ calculated by mean subtraction assuming no net change in roll or pitch over a complete stride. The vertical velocity at each corner of the rectangular back depends on the vertical velocity of the centre of mass, and the roll and pitch angular velocities, and the distance of the corners due to the width and length:

$$V_{\mathrm{zLH,t}} = V_{\mathrm{zCoM,t}} - {}^{w}/_{2}\,\omega_{\mathrm{roll,t}} - {}^{L}/_{2}\,\omega_{\mathrm{pitch,t}}; \tag{S14}$$

$$V_{\mathrm{zRH,t}} = V_{\mathrm{zCoM,t}} + {}^{w}/_{2}\,\omega_{\mathrm{roll,t}} - {}^{L}/_{2}\,\omega_{\mathrm{pitch,t}}; \tag{S15}$$

$$V_{\mathrm{zLF,t}} = V_{\mathrm{zCoM,t}} - {}^{w}/_{2}\,\omega_{\mathrm{roll,t}} + {}^{L}/_{2}\,\omega_{\mathrm{pitch,t}}; \tag{S16}$$

$$V_{\mathrm{zRF,t}} = V_{\mathrm{zCoM,t}} + {}^{w}/_{2}\,\omega_{\mathrm{roll,t}} + {}^{L}/_{2}\,\omega_{\mathrm{pitch,t}}. \tag{S17}$$

The mechanical power at each corner of the rectangular back is due to the vertical force of the respective limb, and the vertical velocity of that corner:

$$P_{\mathrm{LH,t}} = F_{\mathrm{zLH,t}} V_{\mathrm{zLH,t}}; \tag{S18}$$

$$P_{\mathrm{RH,t}} = F_{\mathrm{zRH,t}} V_{\mathrm{zRH,t}}; \tag{S19}$$

$$P_{\mathrm{LF,t}} = F_{\mathrm{zLF,t}} V_{\mathrm{zLF,t}}; \tag{S20}$$

$$P_{\mathrm{RF,t}} = F_{\mathrm{zRF,t}} V_{\mathrm{zRF,t}}. \tag{S21}$$

The overall mechanical work demand comes from the combined positive powers at each corner over the stride; no account is taken of a cost to dissipating energy, nor of elastic storage and recoil; and power transfer between the limbs is assumed to be negligible:

$$W^{+} = \int_0^T \max\left(0, P_{\mathrm{LH,t}}\right) dt + \int_0^T \max\left(0, P_{\mathrm{RH,t}}\right) dt + \int_0^T \max\left(0, P_{\mathrm{LF,t}}\right) dt + \int_0^T \max\left(0, P_{\mathrm{RF,t}}\right) dt. \tag{S22}$$

This can be conveniently expressed in a non-dimensional form by dividing by the idealised work demand of trotting at low duty factor:

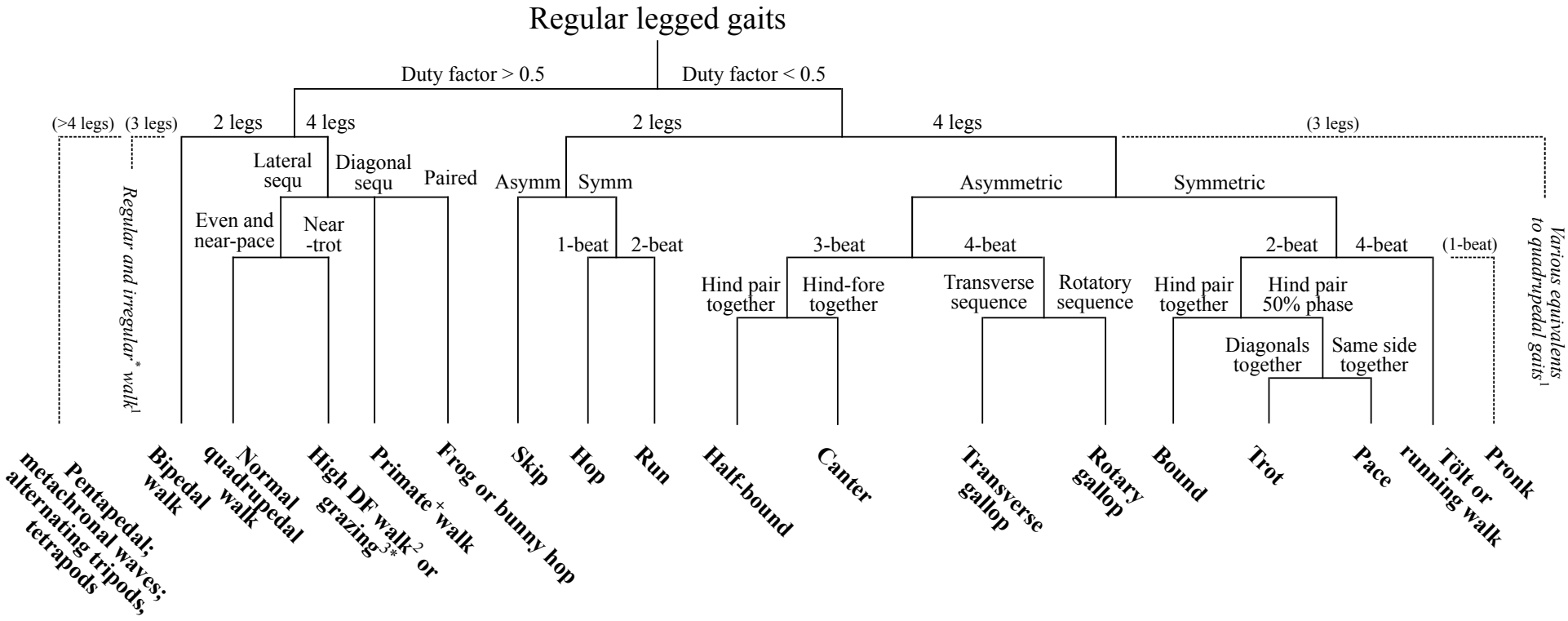
$$\widehat{W^{+}} = \frac{16\, W^{+}}{m g^2 T^2} \tag{S23}$$

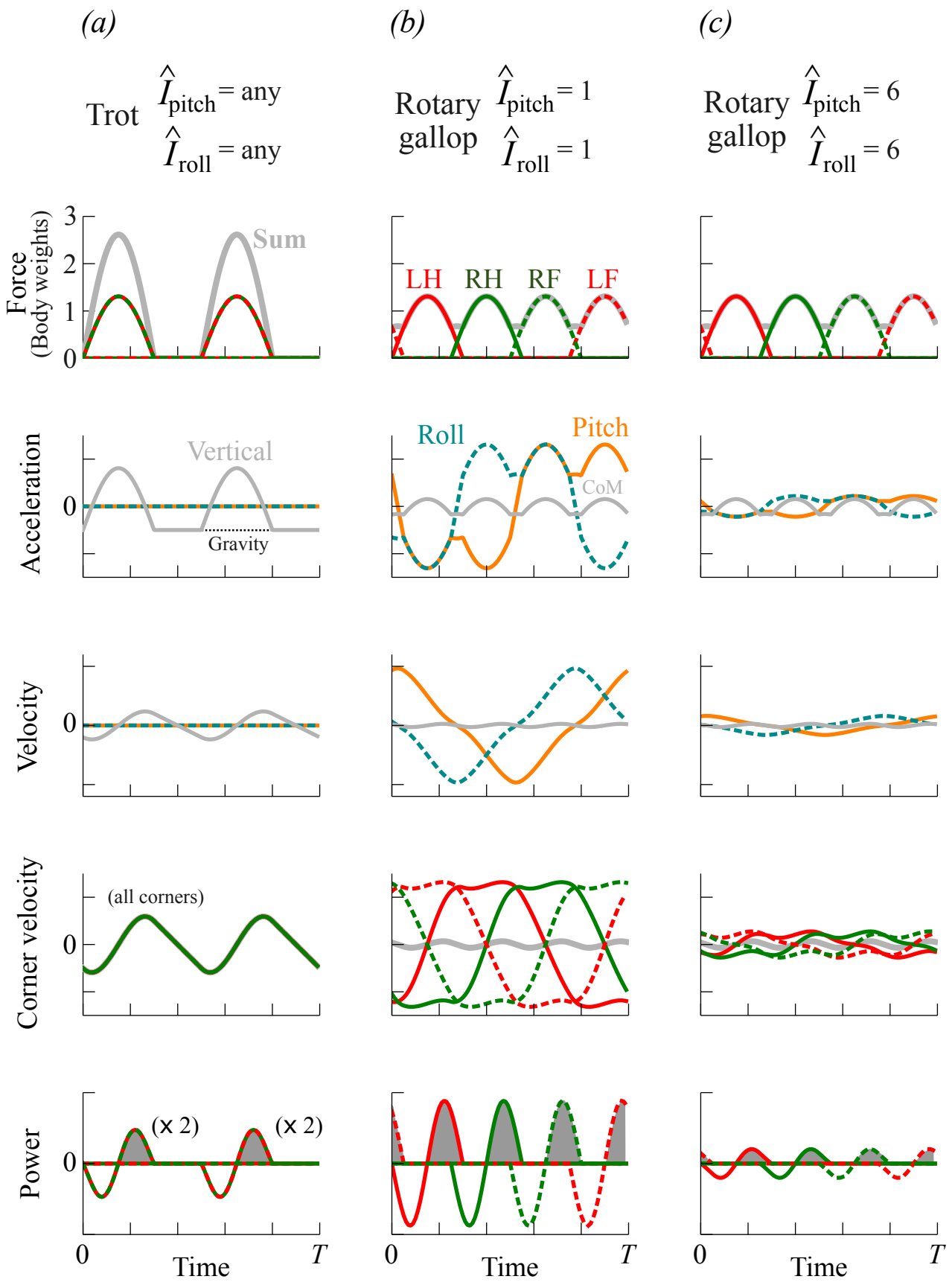
**Extended glossary**

| Term | Definition and context. |
|---|---|
| $a$ | Vertical body dimension: body height. |
| $AR$ | Aspect ratio. |
| $A_z$ | Half-sine vertical force profile amplitude. |
| $b$ | Horizontal body dimension: body width or length. |
| Diagonal sequence | Diagonal sequence of contacts beginning with hind.<br>E.g. diagonal sequence walk starting with LH:<br>LH, RF, RH, LF. |
| Duty Factor | Proportion of stride cycle a foot is in contact with the ground.<br>Higher speeds are generally associated with lower duty factor.<br>Lower duty factors require higher vertical forces to achieve average weight support.<br>Lower duty factors lead to higher energy fluctuations as vertical motions become unavoidable. |
| $F_{z,t}$ | Vertical force due to one leg on the body as a function of time. |
| $g$ | Acceleration due to gravity (gravitational field strength). |
| $I$ | Moment of inertia (second moment of mass).<br>Resistance to rotational acceleration. |
| $I_{diag}$ | Moment of inertia about the roll/pitch diagonal. |
| $I_{pitch}$ | Pitch moment of inertia. |
| $I_{roll}$ | Roll moment of inertia. |
| $\hat{I}_{roll}$ | Non-dimensional roll moment of inertia.<br>Roll moment of inertia normalised with mass and body width. |

| | |
|---|---|
| $\hat{I}_{\text{pitch}}$ | Non-dimensional pitch moment of inertia.<br>Pitch moment of inertia normalised with mass and body length. |
| $KE^{+}_{\text{CoM}}$ | Energy supplied to the centre of mass due to a vertical impulse. |
| $KE^{+}_{\text{rot}}$ | Rotational energy supplied due to a rotational impulse. |
| $J_{\text{CoM}}$ | Vertical impulse applied to the centre of mass.<br>Equivalent to a change in velocity per inertia *m*. |
| $J_{\text{rot}}$ | Rotational impulse.<br>Equivalent to a change in rotational velocity per rotational inertia *I*. |
| $L$ | Body length. |
| Lateral sequence | Lateral sequence of contacts beginning with hind.<br>E.g. lateral sequence walk starting with LH:<br>LH, LF, RH, RF. |
| $L_{\text{step}}$ | Distance travelled by the centre of mass between each foot placement. |
| $L_{\text{foot}}$ | Length of foot. |
| $m$ | Body mass.<br>Resistance to linear acceleration. |
| $N$ | Number of vertical impulses each stride. |
| $N_{\text{rot}}$ | Number of rotational impulses each stride. |
| Phase | Event time expressed as a proportion of the stride cycle.<br>Convention following Hildebrand is that stride cycle begins with a hind contact. |
| Rotary sequence | Rotatory sequence of contacts beginning with hind.<br>E.g. rotary sequence gallop starting with LH:<br>LH, RH, RF, LF. |
| $T$ | Stride period. |

| | |
|---|---|
| $T_{\text{stance}}$ | Stance period. |
| $T_{\text{swing}}$ | Swing period. |
| $t$ | Time through stance. |
| Transverse sequence | Transverse sequence of contacts beginning with hind.<br>E.g. transverse sequence gallop starting with LH:<br>LH, RH, LF, RF. |
| $V_{\text{z}}$ | Vertical velocity. |
| $w$ | Body width. |
| $W^{+}$ | Positive work demand due to vertical force and velocity at all four corners. |
| $\widehat{W^{+}}$ | Positive work demand normalised by the work of impulsive trotting. |
| $W^{+}_{\text{CoM}}$ | Work provided due to all vertical impulses. |
| $W^{+}_{\text{rot}}$ | Work provided due to all rotational impulses. |
| $\beta$ | Symbol for duty factor. |
| $\omega$ | Angular velocity of the body (roll or pitch). |
| $\dot{\omega}$ | Angular acceleration of the body (roll or pitch). |


Regular legged gaits
Duty factor > 0.5
Duty factor < 0.5
(>4 legs)
(3 legs)
2 legs
4 legs
2 legs
4 legs
(3 legs)
Regular and irregular* walk[1]
Lateral sequ
Diagonal sequ
Paired
Even and near-pace
Near -trot
Asymm
Symm
1-beat
2-beat
Asymmetric
Symmetric
3-beat
4-beat
2-beat
4-beat
(1-beat)
Various equivalents to quadrupedal gaits[1]
Hind pair together
Hind-fore together
Transverse sequence
Rotatory sequence
Hind pair together
Hind pair 50% phase
Diagonals together
Same side together
Pentapedal; metachronal waves; alternating tripods, tetrapods
Bipedal walk
Normal quadrupedal walk
High DF walk[2] or grazing[3]*
Primate+ walk
Frog or bunny hop
Skip
Hop
Run
Half-bound
Canter
Transverse gallop
Rotary gallop
Bound
Trot
Pace
Tölt or running walk
Pronk


(a)
(b)
(c)
Trot
$\hat{I}_{pitch}$ = any
$\hat{I}_{roll}$ = any
Rotary gallop
$\hat{I}_{pitch}$ = 1
$\hat{I}_{roll}$ = 1
Rotary gallop
$\hat{I}_{pitch}$ = 6
$\hat{I}_{roll}$ = 6
Force (Body weights)
3
2
1
0
Sum
LH
RH
RF
LF
Acceleration
Vertical
0
Gravity
Roll
Pitch
CoM
Velocity
0
Corner velocity
(all corners)
0
Power
(x 2)
(x 2)
0
0
Time
T
0
Time
T
0
Time
T